(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9585 Roll No. | | | | | | | | | | |

B.Tech.

(SEMESTER-II) THEORY EXAMINATION, 2011-12

ELEMENTARY MATHEMATICS – II

Time : 3 Hours / */ Total Marks : 100*

Note : Attempt questions from each Sections.

Section – A

1. All parts of this question are compulsory. $10 \times 2 = 20$

(a) State the fundamental theorem of algebra.

(b) Solve $5x - 3 < 3x + 1$.

(c) Find n, if $\lfloor n+2 = 2550 \lfloor n$.

(d) Find the total number of ways of answering 5 objective type of questions, each question having 4 choices.

(e) Which term of the A.P.

3, 8, 13, ...

is 248 ?

(f) Find two positive numbers whose difference is 12 and whose A.M. exceeds the G.M. by 2.

(g) Find the value of y so that the line through (3, y) and (2, 7) is parallel to the line through (–1, 4) and (0, 6).

(h) Find the center and radius of

$x^2 + y^2 - 4x + 6y = 12$

(i) Find $\vec{a} \times \vec{b}$, where $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$.

(j) Show that the points A(1, 2, 3), B (3, 4, 7) and C(–3, –2, –5) are collinear.

Section – B

2. Attempt any **three** parts of the following : **3 × 10 = 30**

(a) Solve $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$. Show the graph of the solutions on the number line.

(b) How many different words (may be meaningless) can be formed from the letters of the word "JAIPUR" when the vowels always occupy even places ?

(c) Find the equation of the circle whose radius is 5 and which touches externally the circle $x^2 + y^2 - 2x - 4y = 20$ at the point (5, 5).

(d) Let the position vectors of three points P, Q, R be $\vec{r_1} = 3\hat{i} - 2\hat{j} - \hat{k}$, $\vec{r_2} = \hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{r_3} = 2\hat{i} + \hat{j} - 2\hat{k}$ relative to the origin O. Find the distance of the point P from the plane OQR.

(e) Find the coordinates of the point which divides the line segment joining the points (1, –2, 3) and (3, 4, –5) in the ratio 2 : 3 externally.

Section – C

Attempt any **two** parts from each question. All questions are compulsory. **5 × 10 = 50**

3. (a) Solve the equation $x^2 + 3x + 5 = 0$.

(b) Solve the following system of linear inequalities graphically :

$x + y \geq 5, x - y \leq 3$

(c) Solve the system of inequalities :

$3x - 7 < 5 + x$

$11 - 5x \leq 1$

and represent the solutions on the number line.

4. (a) If $\frac{{}^{15}C_r}{{}^{15}C_{r-1}} = \frac{11}{5}$, find ${}^{8}C_r$.

(b) The third term of an A.P. is 7 and the seventh term is 2 more than 3 times the third term. Find the first term, the common difference and the sum of first 20 terms.

(c) Find the sum to n terms of the series whose n^{th} term is $n(n + 3)$.

5. (a) Find the value of x for which the points $(x, -1)$, (2, 1) and (4, 5) are collinear.

(b) Does the point $\left(\frac{5}{2}, \frac{7}{2}\right)$ lie inside, outside or on the circle $x^2 + y^2 = 25$?

(c) Find the equation of the hyperbola whose foci are $(0, \pm 12)$ and the length of the latus rectum is 36.

6. (a) Find $|\vec{a} - \vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$.

(b) Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$. Show that $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is 45°.

(c) Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 15$.

7. (a) Find the angle between the lines

$$\frac{x-1}{2} = \frac{y-2}{5} = \frac{z+4}{-4} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}.$$

(b) Find the shortest distance between the lines L_1 and L_2 whose vector equations are

$$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

(c) Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and the point (1, 1, 1).

———————